Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/62**

Paper 6 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

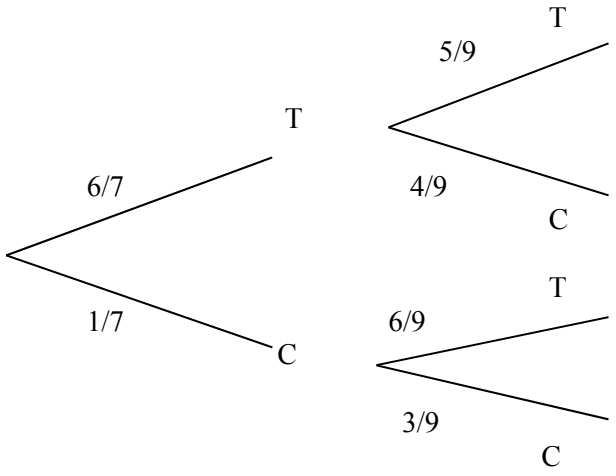
PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$P(S) = \frac{1}{2}$	**B1**	
	$P(T) = \frac{16}{36}\left(\frac{4}{9}\right)$	**B1**	
	$P(S \cap T) = \frac{10}{36}\left(\frac{5}{18}\right)$	**M1**	$P(S \cap T)$ found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	$P(S)\,P(T) \neq P(S \cap T)$ so not independent	**A1**	8/36, 10/36 P(S) × P(T) and $P(S \cap T)$ seen in workings and correct conclusion stated, www
	Alternative method for question 1		
	$P(S) = \frac{1}{2}$	**B1**	
	$P(T) = \frac{16}{36}\left(\frac{4}{9}\right)$	**B1**	
	$P(S \cap T) = \frac{10}{36}\left(\frac{5}{18}\right)$	**M1**	$P(S \cap T)$ found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	$P(S \mid T) = \frac{10}{16}$ or $P(T \mid S) = \frac{10}{18}$ $P(S \mid T) \neq P(S)$ or $P(T \mid S) \neq P(T)$ so not independent	**A1**	**Either** 18/36, 10/16,P(S) and $P(S \mid T)$ seen in workings and correct conclusion stated, www **Or** 16/36, 10/18, P(T) and $P(T \mid S)$ seen in workings and correct conclusion stated, www
		4	

Question	Answer	Marks	Guidance
2	$P(< 28.9) = P\left(z < \frac{28.9-30}{1.5}\right)$	**B1**	Using ± standardising formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$,
	$= P(z < -0.733)$ $= 1 - 0.7682$	**M1**	Appropriate area Φ from standardisation formula P(z <….) in final probability solution, Must be a probability, e.g. 1 – 0.622 is M0
	$= 0.2318$	**A1**	Correct final probability rounding to 0.232. (Only requires M1 not B1 to be awarded
	Number of cartridges is *their* 0.2318×8 $= 1.85$, so 2 (Also accept 1 but not both)	**B1**	**FT** using *their* 4 SF (or better) value, ans. rounded or truncated to integer, no approximation indicated.
		4	

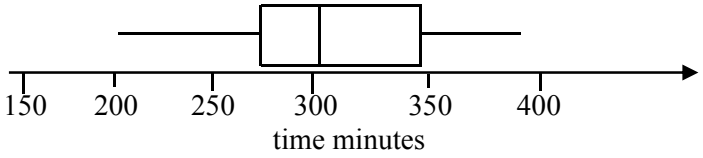
Question	Answer	Marks	Guidance
3(i)	P(at most 7) = 1 – P(8, 9, 10) $= 1 - {}^{10}C8(0.35)^8(0.65)^2 - {}^{10}C_9(0.35)^9(0.65)^1 - (0.35)^{10}$	**M1**	Use of normal approximation M0 Binomial term of form ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	[= 1 – 0.004281 – 0.0005123 – 0.00002759]	**A1**	Correct unsimplified (or individual terms evaluated) answer seen Condone 1 – A + B + C leading to correct solution
	= 0.995	**B1**	B1 **not** dependent on previous marks.
	Alternative method for question 3(i)		
	P(at most 7) = P(0,1,2,3,4,5,6,7)	**M1**	Binomial term of form ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	$= (0.65)^{10} + {}^{10}C1(0.35)^1(0.65)^9 + \ldots + {}^{10}C_7(0.35)^7(0.65)^3$	**A1**	Correct unsimplified answer or individual terms evaluated seen
	= 0.995	**B1**	
		3	
3(ii)	$1 - (0.65)^n > 0.99$ $0.01 > (0.65)^n$	**M1**	Equation or inequality with $(0.65)^n$ **and** 0.01 or $(0.35)^n$ **and** 0.99 only (Note 1 – 0.99 is equivalent to 0.01 etc.)
	$n > 10.69$	**M1**	Solving their $a^n = c$, $0 < a,c < 1$ using logs **or** Trial and Error If answer inappropriate, at least 2 trials are required for Trial and Error M mark
	smallest $n = 11$	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
4	$z = 0.842 = \left(\frac{121-\mu}{\sigma}\right)$ so $0.842\sigma = 121 - \mu$	**B1**	± 0.842 seen but B0 if 1 ± 0.842 oe seen
		M1	One appropriate standardisation equation with a z-value, μ, σ and 121 **or** 102, condone continuity correction. Not 0.158, 0.42,…
	$z = -0.58 = \left(\frac{102-\mu}{\sigma}\right)$ so $-0.58\sigma = 102 - \mu$	**B1**	± 0.58(0) seen but B0 if 1 ± 0.58 oe seen
	Solving	**M1**	Correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable, condone 1 numerical slip
	$\sigma = 13.4$ $\mu = 110$	**A1**	If M0A0 scored (i.e. no algebraic elimination seen), **SC B1** can be awarded for both answers correct Consistent use of σ^2 or $\sqrt{\sigma}$ throughout apply **MR** penalty to A mark or SC B mark.
		5	

Question	Answer	Marks	Guidance
5(i)		**B1**	First pair of branches labels and probs correct (6/7 and 1/7 or rounding to 0.857 and 0.143) (Labelling must be logically…e.g. (T and T) or (T and Not T) would be acceptable)
		B1	Either of second top pair or bottom of branches labels and probs correct
		B1	Both second pairs of branches labels and probs correct. No additional / further branches.
		3	
5(ii)	No of toffees taken (T): 0 | 1 | 2 prob: $\frac{3}{63}$, 0.0476(2) | $\frac{30}{63}$, 0.476(2) | $\frac{30}{63}$, 0.476(2)	**B1**	P(1) correct
		B1	P(0) or P(2) correct
		B1	**FT** Correct values in table, any additional values of T have stated probability of zero. For FT $\Sigma p = 1$,
		3	
5(iii)	$E(X) = \frac{90}{63}$ $(\frac{10}{7})$ (1.43)	**B1**	Not FT
		1	

Question	Answer	Marks	Guidance
5(iv)	$\mathrm{P}(1^{\text{st}}\ \mathrm{C} \mid 2^{\text{nd}}\ \mathrm{T}) = \dfrac{P(C \cap T)}{P(T)} = \dfrac{\frac{1}{7}\times\frac{6}{9}}{\frac{1}{7}\times\frac{6}{9}+\frac{6}{7}\times\frac{5}{9}} = \dfrac{\frac{6}{63}}{\frac{36}{63}}$	**B1**	$\mathrm{P(C} \cap \mathrm{T)}$ attempt seen as numerator of a fraction, consistent with *their* tree diagram or correct
		M1	Summing 2 appropriate two-factor probabilities, consistent with *their* tree diagram or correct seen anywhere
		A1	$\frac{36}{63}$ oe or correct unsimplifed expression seen as numerator or denominator of a fraction
	$\frac{1}{6}$ oe	**A1**	Final answer
		4	

Question	Answer	Marks	Guidance
6(i)	Advantage: comment referring to spread or range or shape	**B1**	Comments referring to quartiles, IQR, Range, median, shape, skewness, data distribution, spread score B1 Any comments with reference to mean or standard deviation or any other 'disadvantage' will score B0 Comments referring to '5-value plot', comparison with another data set, overview or ease of drawing/plotting/reading require an appropriate advantage statement.
	Disadvantage: comment referring to limited data information provided	**B1**	Comments referring to no individual data, no information about the number of values, unable to calculate mean, standard deviation, variance and mode score B1 Any comments with reference to median, shape or any other 'advantage' will score B0 Comments referring to 'size of data set' or 'average' require an appropriate disadvantage statement. Comments referring to outliers are ignored in all cases (as outliers are not in the syllabus content) unless supported by an appropriate advantage / disadvantage statement. If comments not clearly identified, assume first comment is the advantage.
		2	

Question	Answer	Marks	Guidance
6(ii)	Not mean as data skewed by one large value	**B1**	Comment which identifies 768 (or 'a very large number') as the problem. Condone the use of 'outlier'
	Not mode as frequencies all the same	**B1**	Comment which indicates that no mode exists (e.g. all the data is different, there is no repeated number, all the values are different)
	Median	**B1**	Median identified as choice, dependent upon statements for mean and mode being given, even if incorrect or very general.
	SC: Mean is identified as most suitable		
	Not mode as frequencies all the same	**SCB1**	Comment which indicates that no mode exists
	Not median as not all values used	**SCB1**	Comment which indicates limitation of median e.g. median is not in middle of range.
		3	
6(iii)(a)	LQ = 256 or 256.5 Med = 280 UQ = 329 Min 190 max 375  	**B1**	Median, UQ and LQ values seen, may not be identified or identified correctly. (Not read from box plot unless value stated)
		B1	**FT** Median and quartiles plotted in box on graph, linear scale
		B1	Correct end points, whiskers from ends of box but not through box, not at top or bottom of box
		B1	Uniform scale from 190 to 375 (need at least 3 linear identified points min) and labelled 'time' and 'minutes' (can be in title) **No time axis or time axis with no scale attempt,** Max B1B0B0B0
		4	

Question	Answer	Marks	Guidance
6(iii)(b)	IQR = *their* 329 – *their* 256 = 73 or 72.5	**B1**	**FT Must** follow through only from ***their*** stated values (condone if correct quartiles stated here), not reading from graph.
		1	

Question	Answer	Marks	Guidance
7(a)	${}^6C_3 \times {}^3C_2 \times {}^1C_1$	**M1**	${}^6C_a \times {}^{6-a}C_b \times {}^{6-a-b}C_{6-a-b}$ seen oe ${}^{6-a-b}C_{6-a-b}$ can be implied by 1 or omission, condone use of permutations,
	$= 20 \times 3$	**A1**	Any correct method seen no addition/additional scenarios
	$= 60$	**A1**	Correct answer
	Alternative method for question 7(a)		
	$\frac{{}^6P_6}{{}^3P_3 \times {}^2P_2 \times {}^1P_1} = \frac{6!}{3! \times 2!}$	**M1**	${}^6P_6$ / (${}^nP_n$ x k) with $3 \geqslant n > 1$and $6 \geqslant$ k an integer $\geqslant 1$, not 6!/1
		A1	Correct method with no additional terms
	$= 60$	**A1**	Correct answer
		3	
7(b)(i)	$\frac{4!}{3!} \times \frac{3!}{2!} \times 2$	**M1**	A single expression with either 4!/3! × k or 3!/2! × k, k a positive integer seen oe (condone 2 identical expressions being added)
		M1	Correctly multiplying *their* single expression by 2 or 2 identical expressions being added.
	$= 24$	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
7(b)(ii)	Total no of arrangements = $\frac{7!}{2!3!}$ = 420 (A)	**B1**	Accept unsimplified
	No with 2s together = $\frac{6!}{3!}$ = 120 (B)	**B1**	Accept unsimplified
	With 2s not together: *their* (A) – *their* (B)	**M1**	Subtraction indicated, possibly by *their* answer, no additional terms present
	= 300 ways	**A1**	Exact value www
	Alternative method for question 7(b)(ii)		
	3 _ 7 _ 7 _ 7 _ 8 _		
	$\frac{5!}{3!}\times\frac{6\times5}{2}$	**B1**	k x 5! in numerator, k a positive integer
		B1	m x 3! In denominator, m a positive integer
		M1	*Their* 5!/3! multiplied by ${}^6C_2$ only (no additional terms)
	= 300 ways	**A1**	Exact value www
		4	